Eur. Phys. J. C (2008) 56: 429–434
DOI 10.1140/epjc/s10052-008-0670-y

THE EUROPEAN
PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Interacting dark energy with inhomogeneous equation of state

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Received: 17 May 2008 / Published online: 29 July 2008

Abstract We have investigated the model of dark energy interacting with dark matter by choosing inhomogeneous equations of state for the dark energy and a nonlinear interaction term for the underlying interaction. The equations of state have dependencies either on the energy densities, the redshift, the Hubble parameter or the bulk viscosity. We have considered these possibilities and have derived the effective equations of state for the dark energy in each case.

1 Introduction

One of the outstanding developments in astrophysics in the past decade is the discovery that the expansion of the universe is accelerated, supposedly driven by some exotic vacuum energy [1–5]. Surprisingly, the energy density of the vacuum energy is two-third of the critical density ($\Omega_\Lambda \simeq 0.7$) apart from dark matter ($\Omega_{\mathrm{m}} \simeq 0.3$). The astrophysical data suggest that this change in the expansion history of the universe is marginally recent ($z \simeq 0.7$) compared with the age of the universe. The nature and composition of dark energy is still unresolved, but by using thermodynamical considerations it is conjectured that the constituents of dark energy may be massless particles (bosons or fermions) whose collective behavior resembles a kind of radiation fluid with negative pressure. Moreover, the temperature of the universe filled with dark energy will increase as the universe expands [6]. The earliest proposal to explain the recent accelerated expansion was the cosmological constant Λ represented by the equation of state (EoS) $p = -\rho$ (or $\omega = -1$) having a negative pressure. In order to comply with the data, the cosmological constant has to be fine tuned up to 56 to 120 orders of magnitude [7], which requires extreme fine tuning of several cosmological parameters. It also posed the coincidence problem in cosmology (the question of explaining why the vacuum energy came to dominate the universe very recently) [8]. This latter problem is addressed through the notion of a tracker field Q, in which the tracker field rolls down a potential $V(Q)$ according to an attractor-like solution to the equations of motion [9]. But here the field has difficulties reaching $\omega < -0.7$, while current observations favor $\omega < -0.78$ with 95% confidence level [10]. It is shown that a quintessence scalar field coupled with either a dissipative matter field, a Chaplygin gas (CG) or a tachyonic fluid solves the coincidence problem [11]. These problems are alternatively discussed using anthropic principles as well [12]. Several other models have been proposed to explain the cosmic accelerated expansion by introducing a decaying vacuum energy [13, 14], a Cardassian term in the Friedmann–Robertson–Walker (FRW) equations [15], a generalized Chaplygin gas (GCG) [16] and a phantom energy ($\omega < -1$) arising from the violation of energy conditions [17–19]. Another possibility is the 'geometric dark energy' based on the Ricci scalar R represented by $\Re = R/12H^2$, where H is the Hubble parameter [10]. Notice that $\Re > 1/2$ represents accelerated expansion, and $\Re > 1$ gives a super-accelerated expansion of the universe, whereas presently $\Re = 1/2$.

Models based on dark energy interacting with dark matter have been widely investigated [20–31]. These models yield stable scaling solution of the FRW equations at late times of the evolving universe. Moreover, the interacting CG allows the universe to cross the phantom divide (the transition from $\omega > -1$ to $\omega < -1$), which is not permissible in pure CG models. In fact, it is pointed out that a phantom divide (or crossing) is possible only if the cosmic fluids have some interaction [32]. It is possible that this interaction can arise from the time variation of the mass of dark matter particles [33]. It is shown that the cosmic coincidence problem is fairly alleviated in the interacting CG models [34]. This result has been endorsed with interacting dark energy in [35]. There is a report that this interaction is physically observed in the Abell cluster A586, which in fact supports the GCG cosmological model and apparently rules out the ΛCDM model [36]. However, a different investigation of

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the observational $H(z)$ data rules out the occurrence of any such interaction and favors the possibility of either more exotic couplings or no interaction at all [37]. In this context, we have investigated the interaction of the dark energy with dark matter by using a more general interaction term. We have focused on the inhomogeneous EoS for dark energy as these are phenomenologically relevant.

The outline of the paper is as follows. In the next section, we present a general interacting model for our dynamical system. In the third section, we derive the effective EoS for the interacting dark energy by employing different inhomogeneous EoS, having dependencies on various cosmological parameters. Finally, we present our conclusion.

2 The interacting model

We assume the background to be a spatially homogeneous and isotropic FRW spacetime, given by

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right)\right], \quad (1)$$

filled with the two component fluid namely dark energy and dark matter. Here $a(t)$ is the scale factor and $k = -1, 0, 1$ represents the spatially hyperbolic, flat or closed universe, respectively. The corresponding Einstein field equation is

$$H^2 \equiv \left(\frac{\dot a}{a}\right)^2 = \frac{\kappa}{3}\rho - \frac{k}{a^2}, \quad (2)$$

where $\kappa = 8\pi G$ and $\rho = \rho_\Lambda + \rho_\mathrm{m}$. Moreover, the energy conservation for our gravitational system is given by

$$\dot\rho + 3H(\rho + p) = 0, \quad (3)$$

where $p = p_\Lambda = \omega_\Lambda\rho_\Lambda$ and $p_\mathrm{m} = 0$ or $\omega_\mathrm{m} = 0$. We assume a special form of the interaction $Q = \Gamma\rho_\Lambda$ between dark energy and dark matter, where Γ is the decay rate. Then (3) can be subdivided into two parts, corresponding to ρ_Λ and ρ_m as follows:

$$\dot\rho_\Lambda + 3H(1+\omega_\Lambda)\rho_\Lambda = -Q, \quad (4)$$

$$\dot\rho_\mathrm{m} + 3H\rho_\mathrm{m} = Q, \quad (5)$$

respectively. Equations (4) and (5) show that the energy conservation for dark energy and matter would not hold independently if there is interaction between them but would hold globally for the whole interacting system as is manifest in (3). We further define the density ratio r_m, by a form of scaling relation, by $r_\mathrm{m} \equiv \rho_\mathrm{m}/\rho_\Lambda$. To study how this density ratio evolves with time, we differentiate r_m with respect to t:

$$\dot r_\mathrm{m} = \frac{\mathrm{d}r_\mathrm{m}}{\mathrm{d}t} = \frac{\rho_\mathrm{m}}{\rho_\Lambda}\left[\frac{\dot\rho_\mathrm{m}}{\rho_\mathrm{m}} - \frac{\dot\rho_\Lambda}{\rho_\Lambda}\right]. \quad (6)$$

Using (4) and (5) in (6), we get

$$\dot r_\mathrm{m} = 3Hr_\mathrm{m}\left[\omega_\Lambda + \frac{1+r_\mathrm{m}}{r_\mathrm{m}}\frac{\Gamma}{3H}\right]. \quad (7)$$

Furthermore, we define an effective EoS for dark energy and matter by [38]

$$\omega_\Lambda^\mathrm{eff} = \omega_\Lambda + \frac{\Gamma}{3H}, \qquad \omega_\mathrm{m}^\mathrm{eff} = \frac{-1}{r_\mathrm{m}}\frac{\Gamma}{3H}, \quad (8)$$

which also involve the contribution from the interaction between matter and dark energy. Using (8) in (4) and (5), we get

$$\dot\rho_\Lambda + 3H\left(1+\omega_\Lambda^\mathrm{eff}\right)\rho_\Lambda = 0, \quad (9)$$

$$\dot\rho_\mathrm{m} + 3H\left(1+\omega_\mathrm{m}^\mathrm{eff}\right)\rho_\mathrm{m} = 0. \quad (10)$$

From the standard FRW model, the density parameters corresponding to matter and dark energy are defined by

$$\Omega_\mathrm{m} = \frac{\rho_\mathrm{m}}{\rho_\mathrm{cr}}, \qquad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_\mathrm{cr}}. \quad (11)$$

The above parameters are related by $\Omega_\mathrm{m} + \Omega_\Lambda = 1$. Using the definition of r_m, we can write

$$r_\mathrm{m} \equiv \frac{\Omega_\mathrm{m}}{\Omega_\Lambda} = \frac{1-\Omega_\Lambda}{\Omega_\Lambda}. \quad (12)$$

The value of r_m decreases monotonically with expansion and varies very slowly at the present era. Contrary to the noninteracting case, r_m decreases slower when there is an interaction [39]. Using the definition of r_m in (2), we get

$$H^2 = \frac{\kappa}{3}(1+r_\mathrm{m})\rho_\Lambda. \quad (13)$$

Next, we choose the following generalized interaction term:

$$Q = 3Hc\left[\gamma\rho_\mathrm{m} + \beta\rho_\Lambda + \delta(\rho_\mathrm{m}\rho_\Lambda)^{1/2}\right]^n, \quad (14)$$

with the corresponding decay rate

$$\Gamma = 3Hc\left(\beta + \gamma r_\mathrm{m} + \delta\sqrt{r_\mathrm{m}}\right)^n, \quad (15)$$

which follows from $Q = \Gamma\rho_\Lambda^n$, where n, β, γ and δ are constant parameters. The above-mentioned c is the coupling constant. Notice that $c > 0$ yields conversion of dark energy into dark matter, and vice versa if $c < 0$. Note that for $\beta = \gamma = n = 1$ and $\delta = 0$, (14) reduces to the usual linear interaction term [40]. Making use of (14) in (4), the EoS parameter becomes

$$\omega_\Lambda = -1 - \frac{\dot\rho_\Lambda}{3H\rho_\Lambda} - \frac{\Gamma\rho_\Lambda^{n-1}}{3H}. \quad (16)$$

Using (16) in (8), the effective EoS of the dark energy is given by

$$\omega_\Lambda^{\rm eff} = -1 - \frac{\dot\rho_\Lambda}{3H\rho_\Lambda} + \frac{\Gamma}{3H}\left(1-\rho_\Lambda^{n-1}\right). \tag{17}$$

In the forthcoming discussion, we shall determine the effective EoS for dark energy corresponding to various equations of state.

3 Inhomogeneous equations of state for dark energy

The general EoS relating the pressure density p and energy density ρ is given by

$$F(p_\Lambda, \rho_\Lambda) = 0. \tag{18}$$

We shall also consider equations of state depending on either the redshift z, the scale factor $a(t)$ or the bulk viscosity ξ. Note that we are not considering an EoS explicitly containing the time t, as we always have the opportunity to use $a(t) = \varepsilon t^{-\lambda}$, with ε and λ constant parameters.

3.1 Generalized cosmic Chaplygin gas

We now take the EoS of the generalized cosmic Chaplygin gas given by [41]

$$p_\Lambda = -\rho_\Lambda^{-\alpha}\left[C + \left(\rho_\Lambda^{1+\alpha} - C\right)^{-\sigma}\right], \tag{19}$$

where $C = \frac{A}{1+\sigma} - 1$ with $\alpha > 1$ and A constant parameters and $-l < \sigma < 0$, where $l > 1$. This EoS reduces to that of the generalized Chaplygin gas if $\sigma = 0$ and to the Chaplygin gas if furthermore $\alpha = 1$. The motivation to use this EoS is to construct the cosmological models that are stable and free from nonphysical behaviors even when the vacuum fluid behaves like a phantom energy [42].

Using the energy conservation principle, the density evolution is

$$\rho_\Lambda = \left[C + \left(1 + C_1 a^{-3(1+\alpha)(1+\sigma)}\right)^{\frac{1}{1+\sigma}}\right]^{\frac{1}{1+\alpha}}, \tag{20}$$

where C_1 is the constant of integration. We define

$$\Delta_1 \equiv \left(\rho_\Lambda^{1+\alpha} - C\right)^{1+\sigma} - 1 = C_1 a^{-3(1+\alpha)(1+\sigma)}. \tag{21}$$

Making use of (20) and (21) in (17), the effective EoS for the dark energy becomes

$$\omega_\Lambda^{\rm eff} = -1 + \frac{\Delta_1(1+\Delta_1)^{\frac{-\sigma}{1+\sigma}}}{C + (1+\Delta_1)^{\frac{1}{1+\sigma}}} + \frac{\Gamma}{3H}\left(1-\rho_\Lambda^{n-1}\right), \tag{22}$$

where ρ_Λ is determined by (20).

3.2 New generalized Chaplygin gas

Zhang et al. [33] suggested another general form of Chaplygin gas, called the new generalized Chaplygin gas, given by

$$p_\Lambda = \frac{-\tilde A(a)}{\rho_\Lambda^\alpha}, \quad \tilde A(a) = -\omega_\Lambda A a^{-3(1+\omega_\Lambda)(1+\alpha)}. \tag{23}$$

Here α is a constant parameter. This model is dual to the interacting XCDM model, where the X part corresponds to quintessence or X-matter ($\omega_\Lambda < -1/3$).

In this model, the energy density evolves as

$$\rho_\Lambda = \left[A a^{-3(1+\omega_\Lambda)(1+\alpha)} + C_2 a^{-3(1+\alpha)}\right]^{1/(1+\alpha)}, \tag{24}$$

where C_2 is a constant of integration. Thus, using (24) in (17) the effective EoS is given by

$$\omega_\Lambda^{\rm eff} = -1 + \frac{\omega_\Lambda + \Delta_2}{\Delta_2} + \frac{\Gamma}{3H}\left(1-\rho_\Lambda^{n-1}\right), \tag{25}$$

where

$$\Delta_2 \equiv A + C_2 a^{3\omega_\Lambda(1+\alpha)}, \tag{26}$$

and ρ_Λ is determined from (24).

3.3 Generalizing the generalized Chaplygin gas

Sen and Scherrer [43] suggested an EoS for the generalized Chaplygin gas by assuming the constant parameter α to be free, where we have

$$\omega_\Lambda = -\frac{A_{\rm s}}{A_{\rm s} + (1-A_{\rm s})(\frac{a}{a_{\rm o}})^{-3(1+\alpha)}}, \tag{27}$$

where

$$A_{\rm s} = \frac{A}{\rho_{\Lambda_{\rm o}}^{1+\alpha}}. \tag{28}$$

Using (27) one can have various cosmological scenarios: for $0 < A_{\rm s} < 1$ and $\alpha > -1$ we have the standard generalized Chaplygin gas model giving dark matter–dark energy unification. For $A_{\rm s} > 1$ and $\alpha > -1$, it gives the early phantom generalized Chaplygin gas, i.e., it behaves as a phantom energy at early times and behaves like the cosmological constant $\omega_\Lambda = -1$ at late times. For $0 < A_{\rm s} < 1$ and $\alpha < -1$ it represents the transient generalized Chaplygin gas, in which case (27) gives the de Sitter regime at early times and a matter dominated regime at later times.

The density evolution is given by

$$\rho_\Lambda = \rho_{\Lambda_{\rm o}}\left[A_{\rm s} + (1-A_{\rm s})\left(\frac{a}{a_{\rm o}}\right)^{-3(1+\alpha)}\right]^{1/1+\alpha}. \tag{29}$$

The corresponding effective EoS is

$$\omega_\Lambda^{\mathrm{eff}} = -A_s \left(\frac{\rho_\Lambda}{\rho_{\Lambda_o}}\right)^{-(1+\alpha)} + \frac{\Gamma}{3H}\left(1-\rho_\Lambda^{n-1}\right), \tag{30}$$

with ρ_Λ determined by (29).

3.4 Interacting scale factor dependent dark energy

We here take the EoS [44]

$$p_\Lambda = -\rho_\Lambda\left(1 + Aa^\alpha\right). \tag{31}$$

The corresponding density evolution is

$$\rho_\Lambda = C_3 \exp\left(\frac{3Aa^\alpha}{\alpha}\right), \tag{32}$$

with C_3 is constant of integration. The effective EoS is given by

$$\omega_\Lambda^{\mathrm{eff}} = -Aa^\alpha + \frac{\Gamma}{3H}\left(1-\rho_\Lambda^{n-1}\right). \tag{33}$$

3.5 Interacting Hubble parameter dependent dark energy

An interesting EoS depending on the Hubble parameter H is given by [45]

$$p_\Lambda = -\rho_\Lambda + f(\rho_\Lambda) + G(H). \tag{34}$$

The corresponding FRW equation is

$$\dot\rho_\Lambda = -3H\left[f(\rho_\Lambda) + G(H)\right]. \tag{35}$$

Let us choose the following EoS:

$$f(\rho_\Lambda) + G(H) = -A\rho_\Lambda^\alpha - BH^{2\epsilon}, \tag{36}$$

where ϵ is a constant. Using (13) in (36), we get

$$f(\rho_\Lambda) + G(H) = -A\rho_\Lambda^\alpha - B'\rho_\Lambda^\epsilon, \tag{37}$$

where

$$B' \equiv B\left[\frac{\kappa}{3}(1+r_{\mathrm{m}})\right]^\epsilon. \tag{38}$$

Using (35) and (37) in (17), we get

$$\omega_\Lambda^{\mathrm{eff}} = -1 - \left(A\rho_\Lambda^{\alpha-1} + B'\rho_\Lambda^{\epsilon-1}\right) + \frac{\Gamma}{3H}\left(1-\rho_\Lambda^{n-1}\right). \tag{39}$$

Here ρ_Λ is determined from (13).

3.6 Interacting redshift dependent dark energy

We here assume that the dark energy evolves with the redshift parameter z. Hence we take the following linear EoS [46]:

$$\omega(z) = \omega_o + \omega_1 z, \tag{40}$$

where ω_o and ω_1 are constants. This EoS was used to analyze the cosmic microwave background and the matter power spectrum [47]. Equation (40) effectively works for $z < 1$, while $\omega(z) = \omega_o + \omega_1$ holds up till $z \approx 1$. The thermodynamical properties of dark energy have been investigated using (40), and it is deduced that the apparent horizon of the universe may be the boundary of thermodynamical equilibrium for the universe like the event horizon for a black hole [48]. We are interested in the evolution of dark energy (i.e. (40)) in our generalized interacting model. The energy conservation principle gives

$$\rho_\Lambda = C_3 a^{-3(1+\omega_o-\omega_1)} \exp\left(3\omega_1 \frac{a_o}{a}\right), \tag{41}$$

where we have used $z \equiv (a_o/a) - 1$ and C_3 is a constant of integration. Notice that for $\omega_1 = 0$, (41) gives the evolution of the usual dark energy.

Using (41) in (17), we get

$$\omega_\Lambda^{\mathrm{eff}} = \omega_o + \omega_1 z + \frac{\Gamma}{3H}\left(1-\rho_\Lambda^{n-1}\right). \tag{42}$$

The astrophysical data support cosmologies with $\omega_o = -1.25 \pm 0.09$ and $\omega_1 = 1.97^{+0.08}_{-0.01}$ [48]. Also using $0 \le c \le 1$, we see that the right-hand side of (42) becomes negative, i.e. $\omega_\Lambda^{\mathrm{eff}} < 0$, thus supporting the existence of phantom energy.

We now take another EoS [10, 49]:

$$\omega(z) = \omega_o + \omega_1\left(1 - \frac{a}{a_o}\right) = \omega_o + \omega_1\left(\frac{z}{1+z}\right). \tag{43}$$

It avoids the divergent behavior as opposed to (40) and hence is used to parameterize the astrophysical data to higher redshift, up till $z \approx z_{\mathrm{rec}}$. The two constants appearing in (43) are constrained: $-1 \le \omega_o \le -0.434$ and $-0.564 \le \omega_1 \le 0.498$ [50]. Its density evolution is given by

$$\rho_\Lambda = C_4 a^{-3(1+\omega_o+\omega_1)} \exp\left(3\omega_1 \frac{a}{a_o}\right), \tag{44}$$

where C_4 is a constant of integration. Thus, the effective EoS is

$$\omega_\Lambda^{\mathrm{eff}} = \omega_o + \omega_1\left(1 - 3\frac{a}{a_o}\right) + \frac{\Gamma}{3H}\left(1-\rho_\Lambda^{n-1}\right), \tag{45}$$

with ρ_Λ determined from (44).

3.7 Interacting viscous dark energy

The Eckart theory [51] effectively deals with fluids having nonzero viscosities. The term viscosity arises from fluid dynamics, which has two major parts, namely the bulk viscosity ξ and the shear viscosity η. In viscous cosmology, shear viscosities arise in relation to space anisotropy, while the bulk viscosity accounts for the space isotropy [52–54]. The bulk viscous fluid is represented by the Eckart expression $\Pi = -\xi(\rho_\Lambda)u^\mu_{;\mu}$, where u^μ is the four velocity of the viscous fluid. The bulk viscosity is generally taken to be positive to ensure positive entropy production in conformity with the second law of thermodynamics [55]. Its scaling may be represented by $\xi \sim \rho_\Lambda^{-\zeta}$, where ζ is a constant parameter.

The effective pressure containing the isotropic pressure and the viscous stress is given by

$$p_{\mathrm{eff}} = p_\Lambda + \Pi, \tag{46}$$

where $p_\Lambda = \chi/\rho_\Lambda^\alpha$ with $\chi > 0$ and $\Pi = -3H\xi$ in the FRW model. Thus, the energy conservation principle for the bulk viscous fluid becomes

$$\dot\rho_\Lambda + 3H(\rho_\Lambda + p_\Lambda - 3H\xi) = 0. \tag{47}$$

Using (13) in (47) we have

$$\dot\rho_\Lambda + 3H(\rho_\Lambda + p_\Lambda - \xi\Upsilon\sqrt{\rho_\Lambda}) = 0, \tag{48}$$

where $\Upsilon \equiv \sqrt{3\kappa(1+r_{\mathrm{m}})}$. Solving (48) we get

$$a(t) = \left(\frac{1}{C_5}\exp\left[\int \frac{\rho_\Lambda^\alpha\,\mathrm{d}\rho_\Lambda}{\rho_\Lambda^{1+\alpha} - \xi\Upsilon\rho_\Lambda^{\alpha+\frac{1}{2}} + \chi}\right]\right)^{-1/3}, \tag{49}$$

with C_5 is a constant of integration. This equation can be solved exactly by choosing $\xi = \upsilon\rho_\Lambda^{1/2}$ with υ a constant; thus, we have

$$\rho_\Lambda = \left[\frac{(C_5a)^{-3(1-\upsilon\Upsilon)(1+\alpha)} - \chi}{1-\upsilon\Upsilon}\right]^{\frac{1}{1+\alpha}}. \tag{50}$$

Defining

$$\Delta_3 \equiv (C_5a)^{-3(1-\upsilon\Upsilon)(1+\alpha)} = \rho_\Lambda^{1+\alpha}(1-\upsilon\Upsilon) + \chi. \tag{51}$$

Using (50) and (51) in (17), we get the effective EoS of the interacting viscous dark energy:

$$\omega_\Lambda^{\mathrm{eff}} = -1 + (1-\upsilon\Upsilon)\left(\frac{\Delta_3}{\Delta_3-\chi}\right) + \frac{\Gamma}{3H}\left(1-\rho_\Lambda^{n-1}\right). \tag{52}$$

4 Conclusion

In this work, we have determined various effective equations of state for the dark energy having nonzero interaction with the matter in the universe. The dark energy can have dependencies on various cosmological parameters like the Hubble parameter, the redshift, the scale factor, the energy densities or the bulk viscosity. We have considered all such possibilities in our interacting model.

Acknowledgements We would like to thank X. Zhang and S. Odintsov for useful discussions during this work.

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